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On inclusive gluon jet production off the nucleus in perturbative QCD

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Abstract. In the perturbative QCD approach single and double inclusive cross-sections for gluon production off the nucleus are studied by the relevant reggeized gluon diagrams. Various terms corresponding to emission of gluons from the triple pomeron vertex are found. Among them the term derived by Kovchegov and Tuchin emerges as a result of the transition from the diffractive to effective high-energy vertex. However it does not exhaust all the vertex contributions to the inclusive cross-section. In the double inclusive cross-section a contribution violating the naive AGK rules is found in which one gluon is emitted from the vertex and the other from one of the two pomerons below the vertex. But then this contribution is subdominant at high energies and taking it into account seems to be questionable.

1 Introduction

After the equation for the BFKL pomeron in the nucleus (BK equation) had been written, analyzed analytically and solved numerically [1,2], the corresponding inclusive rate of gluon jet production was studied in [3] on the basis of the AGK rules. The obtained cross-sections follow from the cut upper pomeron in the fan diagrams for the whole amplitude (Fig. 1a), the contribution from the cut lower pomerons (Fig. 1b) being cancelled by the AGK rules. Obviously the resulting cross-section is linear in the sum of the pomeron fan diagrams Φ. Some time ago Kovchegov and Tuchin derived the same inclusive rate of gluon jet production in the color dipole formalism [4]. Their result is different from [3] and corresponds to the substitution in the expression obtained in [3]:

$$2\Phi \to 2\Phi - \Phi^2, \quad (1)$$

with a negative quadratic term in Φ. The new term corresponds to emission of gluons from the $3P$ vertex itself, which is not prohibited by the AGK rules (Fig. 1c). This gives us a motivation to reconsider the derivation of the inclusive jet production rate in the fan diagram formalism, which is basically much more transparent in searching for real intermediate states observed as gluon jets, as compared to the color dipole formalism.

Our approach is based on the direct inspection of reggeized gluon diagrams. In the simplest non-trivial example of two scattering centers they can be divided into two contributions: the double pomeron exchange and the triple pomeron interaction with a certain ("diffractive") $3P$ vertex Z. The study of the latter contribution allows one to localize real gluons inside the vertex Z, which can be observed, and thus find the inclusive cross-section corresponding to the emission from the vertex Z. Next we pass from the vertex Z to a different vertex V, which incorporates all the contributions at high energies and describes the splitting of the pomeron in the non-linear BFKL equation. This transition generates a new term which also has the structure of emission from the vertex. This term exactly corresponds to the one introduced by Kovchegov and Tuchin (the KT term).

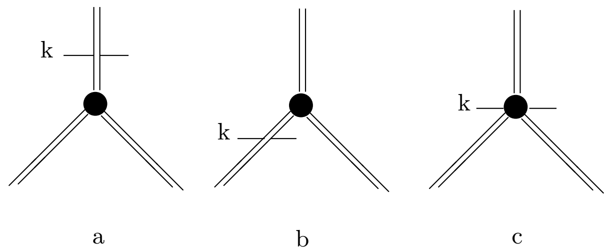

Fig. 1. Pomeron diagrams for the single inclusive cross-section on two centers

As a result, our final inclusive cross-section is found to be much richer than introduced in both our previous paper and by Kovchegov and Tuchin. Apart from the terms derived in these papers it contains several new ones, also quadratic in Φ, and rather complicated in structure. The numerical influence of these new terms will be studied in following publications.

Note that the reggeized gluon diagrams with the vertex Z satisfy the standard AGK rules for their different discontinuities ("cuts"). This can be easily established following the original derivation of the AGK rules in [5]. Indeed for the validity of the AGK rules it is fundamental

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that the coupling vertices are real and do not change when cut in different ways. In the original derivation this property was explained by the fact that the vertex N_ν for the coupling of a particle to ν pomerons could be presented as an integral over the contributions from real intermediate states. So it was irrelevant whether these intermediate particles were cut or not: they were always real. In the diagrams with BFKL pomerons with the triple pomeron vertex Z this property is also true. In fact, various parts of the vertex Z consist of integration kernels $K_{2\to2}$, $K_{2\to3}$ and $K_{2\to4}$ introduced in [6] and connecting 2 gluons with 2, 3 and 4 gluons. All of them are real and contain only real gluons as intermediate states. So they also do not change, whether cut or not. The rest of the derivation of the AGK rules is straightforward and just takes into account that each pomeron may be cut to give $2P$ or not cut to give $-P$ when it lies on the left or right from the cut. This is true irrespective of the nature of the pomeron, the old local one or the BFKL one. Note that this logic works only for the reggeized gluon diagrams as they follow from the perturbation theory, that is, with the triple vertex Z. The transition from Z to the high-energy vertex V introduces virtual gluons into the vertex and thus makes it unfit for the derivation of the AGK rules. This is why we start from the diagrams with the vertex Z

In the final part of the paper we study the double inclusive cross-section on the same lines. Here again we find various contributions corresponding to emission from the vertex. Among them there is a contribution which violates the standard AGK rules. However this violation is much weaker than claimed in [7], where also contributions corresponding to emission from both the upper and lower pomerons were found. Also terms which violate the AGK rules are subdominant at high energies, so that their association with the BK equation is questionable.

2 Emission from pomerons

In this section, also mainly introductory, we reproduce formulas for the single and double inclusive cross-sections which correspond to the emission of gluons from the pomerons. This will serve to fix our normalization and compare with additional contributions coming from emission from the $3P$ vertex.

The scattering amplitude on a nucleus at a given rapidity Y and impact parameter b is represented as

$$\mathcal{A}(Y,b) = 2\mathrm{i}s\int \mathrm{d}^2r\rho(r)\Phi(Y,b,r). \quad (2)$$

Here $\rho(r)$ is the color dipole density in the projectile and Φ is the sum of all pomeron fan diagrams with the vertex $(1/2)V$, where V is the effective high-energy $3P$ vertex introduced in [8–10]. Symbolically

$$\Phi(r) = P(r) - \frac{1}{2}G(r)VP^2 + ..., \quad (3)$$

where $P(r)$ is the pomeron in configuration space, $G(r|r')$ is the corresponding BFKL Green function; G and V are assumed to be operators acting on the pomeron coordinates and rapidities. In the following we always suppress the fixed argument b and often the rapidities when their values are clear. Emission from the pomeron corresponds to "opening" the BFKL chain, which is described by inserting the emission operator (see [11] and also Appendix A). We have

$$V_k(r) = \frac{4\alpha_\mathrm{s}N_c}{k^2}\overleftarrow{\Delta}\,\mathrm{e}^{\mathrm{i}kr}\,\overrightarrow{\Delta}, \quad (4)$$

that is, substituting

$$G_Y(r_1|r_2) \to \int \mathrm{d}^2rG_{Y-y}(r_1|r)V_k(r)G_y(r|r_2), \quad (5)$$

where we indicated the rapidities as subindices.

In this way we find the single inclusive cross-section corresponding to the emission from the uppermost pomeron in Φ (at fixed b):

$$J(y,k) \equiv \frac{(2\pi)^3\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}^2k\mathrm{d}^2b} = 2\int \mathrm{d}^2r_1\mathrm{d}^2r\rho(r_1)G_{Y-y}(r_1|r)V_k(r)\Phi_y(r). \quad (6)$$

The AGK rules tell us that emission from lower pomerons in Φ does not give any contribution. Note that the same cancellation has been found in the dipole picture in [4].

The double inclusive cross-section has two contributions from pomerons. The first one is the double emission from the uppermost pomeron (Fig. 2a)

$$J_1(y_1,k;y_2,l)) = 2\int \mathrm{d}^2r_1\mathrm{d}^2r\mathrm{d}^2r' \quad (7)$$

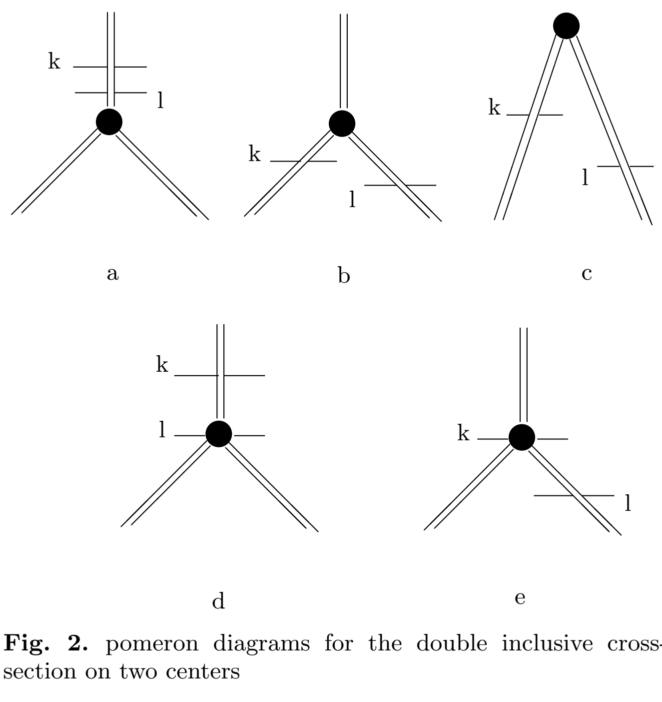


Fig. 2. pomeron diagrams for the double inclusive cross-section on two centers

$$\times\rho(r_1)G_{Y-y_1}(r_1|r')V_k(r')G_{y_1-y_2}(r'|r)V_l(r)\Phi_{y_2}(r).$$

The second contribution comes as emissions from the two pomerons immediately below the vertex (Fig. 2b)

$$J_2(y_1,k;y_2,l)) = -\int_{y_1}^{Y}\mathrm{d}y\int \mathrm{d}^2r\mathrm{d}^2r'\prod_{i=1}^{4}\mathrm{d}^2r_i\rho(r_1)G_{Y-y}(r_1|r_2) \times Z(r_2|r_3,r_4)G_{y-y_1}(r_3|r)V_k(r) \times\Phi_{y_1}(r)G_{y-y_2}(r_4|r')V_l(r')\Phi_{y_2}(r'), \quad (8)$$

where $Z(r_2|r_3,r_4)$ is the $3P$ vertex in the coordinate space for the forward direction.

3 Single inclusive cross-section: emission from vertex Z

Contributions presented in the previous sections ("naive") were introduced and studied in [3]. However the analysis of the amplitude with 4 reggeized gluons shows that apart from real gluons in the pomeron chains there appear new real gluons in the process of changing the number of reggeized gluons or interactions between different pomerons. So one may expect an additional contribution to the inclusive cross-sections coming from the observation of real gluons inside the $3P$ vertex. To this end it is sufficient to study the scattering on two centers (nucleons), which corresponds to the second term in the series (3). Then we can investigate emission from the vertex as it follows from inspection of the amplitude for 4 reggeized gluons obtained in the high-color limit in [12]. Our main idea is that the results for the amplitude should be consistent with the AGK rules, which tell that the relation between the diffractively cut, double cut and single cut amplitude is $1 : 2 : -4$. As mentioned, the validity of these rules is actually based only on the fact that the $3P$ vertex is real and does not change for various cuttings, which is true in our case. Knowing this and inspecting particular contributions we then can establish whether and how the $3P$ vertex is cut and from this find the corresponding contribution to the inclusive cross-section. Note that the amplitude D_4 for 4 reggeized gluons introduced in [9] and studied in [12] and below is normalized to the diffractive discontinuity for two identical centers. The amplitude corresponding to the scattering on two centers is therefore obtained as $-(1/2)D_4$. This factor $-(1/2)$ appears as a coefficient before the vertex in the summation of pomeron fans in (3) and has to be remembered when translating our formulas from [12] into the ones for the inclusive cross-sections.

3.1 Transitions $2 \to 4$ gluons

We suppress the pomerons connecting the amplitude D_4 to the two scattering centers ("attached from below"). For the transition from 2 to 4 gluons we have a single contribution to the amplitude, which we then write as

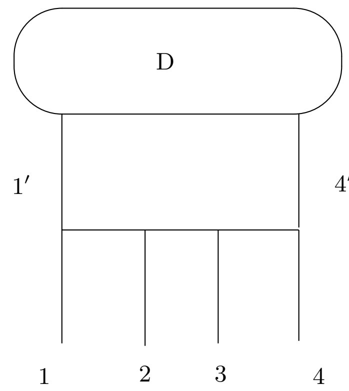


Fig. 3. Transitions from 2 to 4 gluons

$$D_{2\to4}^{(0)} = -g^4N_cW(1,23,4|1',4')\otimes D(1'). \quad (9)$$

We use the notation from [12]. Momenta are denoted by the number of the gluons which carry them: $1 \equiv k_1$ etc. The notation 12 means the sum of momenta 1 and 2. The function W is the momentum part of the Bartels kernel $K_{2\to3}$:

$$W(k_1,k_2,k_3|q_1,q_3) = \frac{(k_1+k_2+k_3)^2}{q_1^2q_3^2} + \frac{k_2^2}{(k_1-q_1)^2(k_3-q_3)^2} - \frac{(k_1+k_2)^2}{q_1^2(k_3-q_3)^2} - \frac{(k_2+k_3)^2}{q_3^2(k_1-q_1)^2}. \quad (10)$$

It conserves the momentum so that $k_1+k_2+k_3 = q_1+q_3$. It is assumed that the pairs of final gluons 12 and 34 are colorless and are to be coupled to two final pomerons. $D(1)$ is the initial pomeron (amputated). The symbol $\otimes$ in this section means integration over $1'$ with the weight $(2\pi)^{-3}$. Diagrammatically the contribution (9) is shown in Fig. 3. One has to understand that in the whole amplitude the gluon pairs 12 and 34 are to be represented by the standard BFKL ladders.

Now we pass to the determination of the inclusive cross-section corresponding to a real gluon inside the $3P$ vertex.

According to the AGK rules, with the adopted normalization, the diffractive contribution is 1/2 of (9). From Fig. 3 we observe that in the diffractive contribution the central real gluon is cut in W and the two side ones are uncut. To find the inclusive cross-section corresponding to observation of the cut central gluon we have to fix its momentum. Therefore the contribution to the inclusive cross-section to observe a gluon of momentum k coming from the vertex will be

$$I_{2\to4}^{\mathrm{dif}} = -g^4N_cW(1,23,4|12k,34-k)D(12k). \quad (11)$$

Of course there will be no integration, so that $\otimes$ passes into a simple product. The factor $(2\pi)^{-3}$ is assumed to be included in I. We have also taken into account factor 2 in (2).

The double cut contribution is twice (9) according to the AGK rules. Since both lower pomerons are now cut, we conclude from Fig. 3 that gluons on both sides in W are cut and the central gluon is uncut. To find the corresponding inclusive one from the vertex we have to fix the momentum of one of the side gluons. The two possibilities give the same contribution so that finally

$$I^{\rm double}_{2\to 4} = -4g^4 N_c W(1, 23, 4|1k, 4-k) D(1k). \quad (12)$$

The single cut contributions give (9) multiplied by (-2). In them only one of the side gluons in W is cut, the other and the central left uncut. So to find the inclusive cross-section we have just to fix the momentum of the cut gluon. We find

$$I^{\rm single}_{2\to 4} = 4g^4 N_c W(1, 23, 4|1k, 4-k) D(1k). \quad (13)$$

As a result, the double and single cut contributions cancel and we are left with the diffractive contribution (11), which gives the total inclusive cross-section from the vertex from transitions $2 \to 4$

3.2 Transitions $3 \to 4$ gluons

According to [12] the contribution from transitions $3 \to 4$ gluons to the amplitude is given by

$$D^{(0)}_{3\to 4} = g^3\sqrt{2N_c}\Big\{W(1,2,3|1',3') \otimes D_3^{(134)}(1',3',4) \\ -W(1,2,4|1',4') \otimes D_3^{(134)}(1',3,4') \\ +W(2,3,4|2',4') \otimes D_3^{(124)}(1,2',4') \\ -W(1,3,4|1',4') \otimes D_3^{(124)}(1',2,4')\Big\}, \quad (14)$$

where

$$D_3^{(123)}(1,2,3) = g\sqrt{\frac{N_c}{8}}\,(D(2) - D(1) - D(3)). \quad (15)$$

Let us consider the first term in the sum (14) as an example. It is illustrated in Fig. 4. Again one has to imagine that the pairs of legs 12 and 34 actually are the beginning of two BFKL ladders corresponding to lower pomerons.

We again start with the diffractive cut. Obviously it corresponds to the cut right gluon in W, while the left one stays uncut. The contribution to the inclusive cross-section from the vertex will be

$$I^{\rm dif,1}_{3\to 4} = g^3\sqrt{2N_c} W(1,2,3|12k, 3-k) \\ \otimes D_3^{(134)}(12k, 3-k, 4). \quad (16)$$

In the double cut amplitude it is the left gluon in W which will be cut, while the right one will be uncut. Taking into account the factor 2 from the AGK rules we find the contribution from the first term in (14):

$$I^{\rm double,1}_{3\to 4} = 2g^3\sqrt{2N_c} W(1,2,3|1k, 23-k) \\ \otimes D_3^{(134)}(1k, 23-k, 4). \quad (17)$$

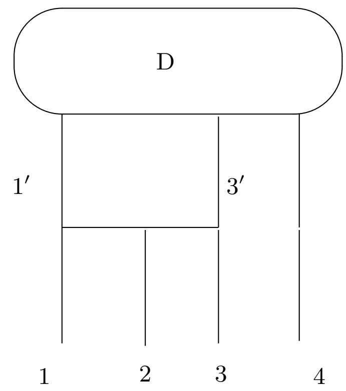


Fig. 4. Transitions from 3 to 4 gluons

Finally in the once cut amplitude the contribution will come only from the half of all terms, namely from those in which the lower pomeron 12 is cut. If the pomeron 34 is cut there is no real gluon in the vertex. So the total factor is (-2) and the observed gluon from the vertex is the left one, while the right one is unobserved. The contribution to the inclusive cross-section from the vertex will be

$$I^{\rm single,1}_{3\to 4} = -2g^3\sqrt{2N_c} W(1,2,3|1k, 23-k) \\ \otimes D_3^{(134)}(1k, 23-k, 4). \quad (18)$$

As for transitions $2 \to 4$ the contributions from the double cut and single cut amplitudes cancel and we are left with only the diffractive contribution (16). The same result holds for the rest of the terms in (14).

3.3 Transitions $4 \to 4$ gluons

The contribution to the amplitude from transitions $4 \to 4$ gluons is given by [12]

$$D^{(0)}_{4\to 4} = g^2\,(U_{23} + U_{14} - U_{13} - U_{24}) \\ \otimes \left(D_4^{(1234)} - D_4^{(2134)}\right), \quad (19)$$

where

$$D_4^{(1234)}(1,2,3,4) = \frac{1}{4} g^2 N_c\,(D(1) + D(4) - D(14)) \quad (20)$$

and

$$D_4^{(2134)}(1,2,3,4) \\ = \frac{1}{4} g^2 N_c\,(D(2) + D(3) - D(12) - D(13)). \quad (21)$$

Here $U_{23} = U(2,3|2',3')$ is the BFKL interaction acting between the gluons 2 and 3:

$$U(k_1,k_2|q_1,q_2) = \frac{k_1^2 q_2^2 + k_2^2 q_1^2}{q_1^2 q_2^2 (k_1 - q_1)^2} - \frac{(k_1+k_2)^2}{q_1^2 q_2^2}, \quad (22)$$

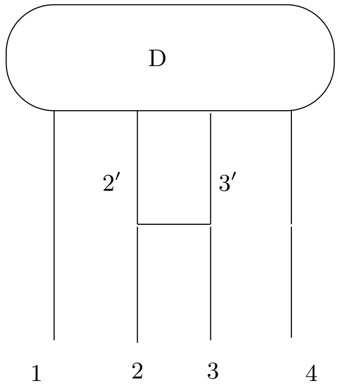


Fig. 5. Transitions from 4 to 4 gluons

with $k_1 + k_2 = q_1 + q_2$.

As an example we consider the term with U_{23} illustrated in Fig. 5. Obviously both single and double cuts through the lower pomerons do not pass through the interaction U_{23}, which acts between the two pomerons. So the only contribution will come from the diffractive cut. Fixing the observed gluon momentum we find the contribution to the inclusive cross-section from the vertex as

$$I_{4\to4}^{(23)} = I_{4\to4}^{\mathrm{dif},(23)}$$
$$= g^2 U(2,3|2k,3-k) \tag{23}$$
$$\times \left(D_4^{(1234)}(1,2k,3-k,4) - D_4^{(2134)}(1,2k,3-k,4)\right).$$

The total contribution is the sum of this term with the ones coming from the interactions U_{14}, U_{13} and U_{24} (with their respective signs) which are treated in a similar manner.

3.4 Total contribution

We have found that the whole contribution $I^{(Z)}$ to the inclusive cross-section from the vertex Z only comes from the diffractive cut. Integration over the observed gluon momentum obviously gives the modulus of the total amplitude

$$\int \frac{\mathrm{d}^2 k}{(2\pi)^3} I^{(Z)}(k) = Z \otimes D. \tag{24}$$

To this expression one should attach two lower pomerons, which has been always assumed implicitly. Then (24) means that the integrated contribution of the emission from the vertex Z is equal to the total high-mass diffractive cross-section.

Summing (11), 4 terms of the type (16) and 4 terms of the type (23), using the symmetry in the gluons 12 and 34 and attaching two lower forward pomeron fans $\Phi(1)$ and $\Phi(3)$ with $12 = 34 = 0$ we find

$$I^{(Z)}(k) = g^2 N_c \int \frac{\mathrm{d}^2 k_1}{(2\pi)^2}\frac{\mathrm{d}^2 k_3}{(2\pi)^2}\Phi(1)\Phi(3)$$
$$\times \{W(1,2,3|12k,3-k)\,(D(3-k)-D(4))$$
$$+W(2,3,4|2k,34-k)\,(D(2k)-D(1))$$
$$-W(1,23,4|12k,34-k)D(12k) \tag{25}$$
$$+\,U(2,3|2k,3-k)\,(D(13-k)-D(14))\}\,.$$

Here we have put $P \to (1/g^2)\Phi$ for the lower pomerons and $D \to g^2 D$ for the upper one in accordance with their definitions in [2]. More explicit expressions for the inclusive cross-section $I^{(Z)}(k)$ can be found in the Appendix B.

4 Transformation to the vertex V

The total inclusive cross-section for gluon production on two centers is the sum of (25) and a contribution from the cut upper pomeron in the triple pomeron diagram with vertex Z:

$$I^{\mathrm{tot}}(k) = I^{(Z)}(k) + J^{(Z)}(k), \tag{26}$$

where we denoted $J^{(Z)}$ the standard contribution (6) obtained by cutting the upper pomeron for two scattering centers with vertex Z:

$$J^{(Z)}(k) = -\frac{g^2 N_c}{2\pi}\int \frac{\mathrm{d}^2 k_1}{(2\pi)^2}\frac{\mathrm{d}^2 k_3}{(2\pi)^2}\frac{\mathrm{d}^2 k_1'}{(2\pi)^2}\frac{\mathrm{d}^2 k_5}{(2\pi)^2}$$
$$\times P(1)P(3)Z(1,-1,3,-3|1',-1')$$
$$\times \tilde{G}(1'|5)U(5|5k)D(5k). \tag{27}$$

Here $\tilde{G}(1'|5)$ is the forward BFKL Green function, amputated from the left (that is, without the left factor $k_1'^{-4}$), satisfying the equation

$$(j-1-2\omega(1)-U)\tilde{G}(1|1') = (2\pi)^2\delta^2(1-1'), \tag{28}$$

where ω is the gluon trajectory. We denote

$$B(1,2,3,4|5)$$
$$= G_4(1,2,3,4|1',2',3',4') \otimes Z(1',2',3',4'|5',-5')$$
$$\times \otimes \tilde{G}(5'|5), \tag{29}$$

where $\otimes$ means integration over all repeated momenta with weight $(2\pi)^{-2}$, $1234 = 0$, and $G_4(1,2,3,4|1',2',3',4') = G(1,2|1',2')G(3,4|3',4')$. In fact also $12 = 1'2' = 34 = 3'4' = 0$; however, this will be irrelevant for the time being. Obviously B satisfies the equation

$$S_4 B = Z \otimes \tilde{G} + g^2 N_c (U_{12} + U_{34}) B. \tag{30}$$

This is actually the same equation as in our derivation of the vertex V in [10], with the difference that D is substituted by $\tilde{G}$. The latter is also a solution of the BFKL equation although with a different inhomogeneous term. So we can repeat our whole derivation of the vertex V in [10] substituting in D_{40}

$$D_{20}(q) \to \tilde{G}^{(0)}(q|k_5) = (2\pi)^2\delta^2(q-k_5) \tag{31}$$

and separating from B a “reggeized term”

$$B^R = D_{40}\left(D_{20}(q) \to \tilde{G}(q|5)\right). \tag{32}$$

As a result, we shall get

$$\begin{aligned} B = & B^{\mathrm{R}} \\ & + G_4(1,2,3,4|1',2',3',4') \\ & \quad \otimes \{V(1',2',3',4'|5',-5') \otimes G_2(5',5) \\ & \quad - D_{40}\left(D_{20}(q) \to \tilde{G}^{(0)}(q|5)\right)\Big\}\,. \end{aligned} \tag{33}$$

Putting into (27) these three terms will give three contributions to the inclusive cross-section, which have transparent interpretations. The reggeized term B^R attached to the cut interaction $U(5|5k)$ and the upper amputated pomeron $D(5k)$ will give the contribution corresponding to "opening" the reggeized term for the amplitude D_4^{R}. This term is subdominant at high rapidities and not taken into account in the non-linear BFKL equation. The second term will generate the standard contribution from the triple P interaction with vertex V following from the AGK rules and coming from the upper pomeron. The third term in (33) is independent of rapidity and putting it into (27) will give a contribution $I_1(k)$ which has the meaning of emission from the vertex.

Explicitly we have

$$\begin{aligned} & D_{40}\left(D_{20}(q) \to \tilde{G}^{(0)}(q|5)\right) \\ & = \frac{1}{2}g^2(2\pi)^2\left\{\sum_{i=1}^{4}\delta^2(i-5) - \sum_{i=2}^{4}\delta^2(1i-5)\right\}. \end{aligned} \tag{34}$$

From the first term in the first sum we shall have a contribution

$$\begin{aligned} I_1^{(1)}(k) = & \frac{1}{2}g^2(2\pi)^2\frac{g^2N_c}{2\pi}\int\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\frac{\mathrm{d}^2k_5}{(2\pi)^2} \\ & \times P(1)P(3)U(5|5k)D(5k)\delta^2(1-5). \end{aligned} \tag{35}$$

Integration over k_3 puts the gluons in the pomeron $P(3)$ at the same point in the configuration space and makes the total contribution vanish. The same argument is true for all terms in the first sum.

The first term in the second sum gives

$$\begin{aligned} I_1^{(2)}(k) = & -\frac{1}{2}g^2(2\pi)^2\frac{g^2N_c}{2\pi}\int\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\frac{\mathrm{d}^2k_5}{(2\pi)^2} \\ & \times P(1)P(3)U(5|5k)D(5k)\delta^2(12-5). \end{aligned} \tag{36}$$

However, $12=0$, so that now integrations over k_1 and k_3 will put gluons in both the lower pomerons at the same point in configuration space. So this term also vanishes.

We are left with the two last terms in the second sum. They give equal contributions and their sum is

$$\begin{aligned} & I_1^{(3)}(k) \\ & = -g^2(2\pi)^2\frac{g^2N_c}{2\pi}\int\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\frac{\mathrm{d}^2k_5}{(2\pi)^2} \\ & \quad \times P(1)P(3)U(5|5k)D(5k)\delta^2(13-5). \end{aligned} \tag{37}$$

Using the explicit expression

$$U(5,6) = 2\frac{k_5^2}{k_6^2(k_5-k_6)^2} \tag{38}$$

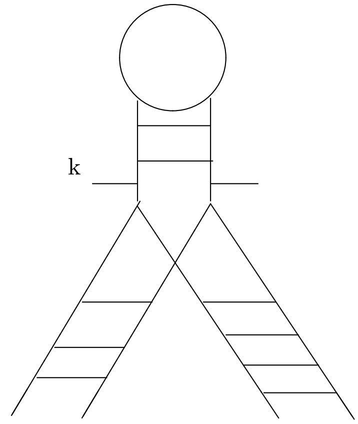


Fig. 6. The Kovchegov–Tuchin term for the inclusive cross-section

and passing to the non-amputated upper pomeron

$$D(6) = k_6^4P(6) \tag{39}$$

we rewrite the contribution (37) in the final form

$$\begin{aligned} & I_1^{(3)}(k) \\ & = -g^2\frac{g^2N_c}{\pi}\frac{1}{k^2}\int\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2} \\ & \quad \times P(1)P(3)(k_1+k_3)^2(k_1+k_3+k)^2P(13k). \end{aligned} \tag{40}$$

Passing to the configuration space one can see that this contribution is exactly the additional term derived by Kovchegov and Tuchin in the dipole formalism in [4]. Putting $P \to g^2P$ for the upper pomeron and $P \to (1/g^2)\Phi$ for the lower ones we find

$$\begin{aligned} I_1^{(3)}(k) \equiv & I^{(\mathrm{KT})}(k) \\ & = -\frac{4\alpha_{\mathrm{s}}N_c}{k^2}\int \mathrm{d}^2r\mathrm{e}^{\mathrm{i}kr}\Delta P(r)\Delta\Phi^2(r). \end{aligned} \tag{41}$$

Thus the Kovchegov–Tuchin (KT) term is just the difference between the standard AGK contributions obtained by cutting the upper pomeron in the triple P diagram with vertices Z and V. The interpretation of this term in terms of reggeized gluon diagrams is illustrated in Fig. 6.

However from our derivation it follows that the total contribution from the vertex V has to additionally include the part $I^{(Z)}$ which appears in the direct calculation with vertex Z:

$$I^{(V)} = I^{(\mathrm{KT})} + I^{(Z)}. \tag{42}$$

5 Double inclusive cross-section

Now we pass to the double inclusive cross-section $I(k,l)$ for the production of two gluon jets with rapidities and momenta y_1, k and y_2, l with $y_1 \gg 1$, $y_2 \gg 1$ and $Y \gg y_1 \gg y_2$ where Y is the overall rapidity. In the following we shall often suppress the rapidities y_1 and

y_2 associating them with the given observed gluon momenta. By the AGK rules and without emission from the vertex the double inclusive cross-section is described by the first three diagrams shown in Fig. 2. Allowed by the AGK rules, emission from the vertex adds to them a new diagram, Fig. 2d. Note that in contrast to the single inclusive cross-section different contributions to the double inclusive cross-section behave differently at large rapidities. Two emissions from the upper pomeron (Fig. 2a) and one from the upper pomeron and the other from the vertex (Fig. 2d) generate cross-sections which grow as $\exp \Delta(Y + y_2)$, whereas emissions from the double pomeron exchange (Fig. 2c) or the two lower pomerons (Fig. 2b) give contributions which grow as $\exp 2\Delta Y$ and clearly dominate. So taking into account the former contributions (or any with a similar high-rapidity behavior) in the summation of pomeron fans is not justified, as this summation is standardly performed in the high-rapidity limit. This problem is not academic, since as we shall see, the analysis of the reggeized gluon diagrams will produce new contributions and among them a contribution corresponding to the diagram Fig. 2e prohibited by the AGK rules and with the emission from the vertex different from Fig. 2d. However, all of them are subdominant relative to the emission from the double pomeron exchange or the two lower pomerons (Fig. 2a,c) and strictly speaking have to be neglected.

5.1 Emission from the vertex

As with the single inclusive cross-section we begin with the study of emission from the vertex itself. Since the two observed gluons are assumed to have widely different rapidities, only one of them may be emitted from the vertex. Obviously there may be two possibilities. Either it is the slowest gluon l which is emitted from the vertex, the quicker one k emitted from the upper pomeron, or the quicker gluon k is emitted from the vertex and the slow l from one of the lower pomerons. We start from the first possibility.

If the gluon l is emitted from the vertex and the gluon k is emitted from the upper pomeron (Fig. 2d) then the corresponding cross-section can be found in a trivial manner. In fact, in the diagrams for the single inclusive cross-section the upper pomeron is always cut. So to find the cross-section corresponding to Fig. 2d all we have to do is to take the single inclusive emission from the vertex $I^{(Z)}(l)$, (25), and “open” the upper pomeron to describe emission of the gluon k from it, that is, substitute the upper pomeron $D(q)$ according to (5), which in the momentun space means

$$D(q) \to G(q, q_1) \otimes V_k(q_1, q_2) \frac{D(q_2)}{q_2^4}, \tag{43}$$

where

$$V_k(q_1, q_2) = \frac{4\alpha_c N_c}{K^2} q_1^2 q_2^2 (2\pi)^2 \delta^2(q_1 - q_2 - k). \tag{44}$$

For the diagram Fig. 2d this means inserting into the vertex the same emission operator which acts in the single inclusive cross-section corresponding to Fig. 1c.

Now we consider the second possibility when one (the quickest) gluon is emitted from the vertex and the other from the lower pomeron (Fig. 2e). This time we have to return to the diagrams for reggeized gluon propagation shown in Figs. 3–5. In contrast to the single inclusive cross-section studied in Sect. 2, now we must take into account that at least one of the pomerons attached to gluons 12 or 34 (that from which the second gluon is emitted) has to be cut.

As before we start from transitions from 2 to 4 gluons. Obviously the diffractive cut passing through the center does not give any contribution, since none of the lower pomerons is cut. The double cut, passing through the two pomerons, will give two contributions, corresponding to the observation of the left real gluon in the vertex

$$\begin{aligned} -2g^4 N_c \, (&P(1', 2')V_l G(1', 2'|1, 2)P(3, 4) \\ &+ P(1, 2)P(3', 4')V_l G(3', 4'|3, 4)) \\ &\otimes W(1, 23, 4|1k, 234 - k)D(1k) \end{aligned} \tag{45}$$

or the right real gluon in the vertex

$$\begin{aligned} -2g^4 N_c \, (&P(1', 2')V_l G(1', 2'|1, 2)P(3, 4) \\ &+ P(1, 2)P(3', 4')V_l G(3', 4'|3, 4)) \\ &\otimes W(1, 23, 4|123k, 4 - k)D(4 - k). \end{aligned} \tag{46}$$

In the single cut contributions the second gluon can only be emitted from the cut lower pomeron, the left one:

$$\begin{aligned} 4g^4 N_c &P(1', 2')V_l G(1', 2'|1, 2)P(3, 4) \\ &\otimes W(1, 23, 4|1k, 234 - k)D(1k), \end{aligned} \tag{47}$$

or the right one:

$$\begin{aligned} 4g^4 N_c &P(1, 2)P(3', 4')V_l G(3', 4'|3.4) \\ &\otimes W(1, 23, 4|123k, 4 - k)D(4 - k). \end{aligned} \tag{48}$$

Due to symmetry under the interchange of gluons $(12) \leftrightarrow (34)$ contributions from the double and single cuts cancel. So transitions from 2 to 4 gluons give no contribution to the inclusive cross-section corresponding to the diagram Fig. 2e.

The same is true for transitions from 4 to 4 gluons of the type shown in Fig. 5. The real gluon inside the vertex corresponds to only the diffractive cut when the lower pomerons are both uncut.

We are left with transitions from 3 to 4 gluons, see (14), the first term of which is illustrated in Fig. 4. Let us start from this term. Again the diffractive cut gives no contribution. From the double cut we have a contribution corresponding to the observed left real gluon in the vertex:

$$\begin{aligned} 2g^3 \sqrt{2N_c} \, (&P(1', 2')V_l G(1', 2'|1, 2)P(3, 4) \\ &+ P(1, 2)P(3', 4')V_l G(3', 4'|3.4)) \\ &\otimes W(1, 2, 3|1k, 23 - k)D_3^{(134)}(1k, 23 - k, 4). \end{aligned} \tag{49}$$

The single cut contribution comes only from the cut pomeron 12:

$$-4g^3\sqrt{2N_c}P(1',2')V_lG(1',2'|1,2)P(3,4) \otimes W(1,2,3|1k,23-k)D_3^{(134)}(1k,23-k,4). \quad (50)$$

In the sum we get

$$2g^3\sqrt{2N_c}\,(P(1,2)P(3',4')V_lG(3',4'|3,4) - P(1',2')V_lG(1',2'|1,2)P(3,4)) \otimes W(1,2,3|1k,23-k)D_3^{(134)}(1k,23-k,4). \quad (51)$$

Due to asymmetry between the contributions from the same 12 or different 34 lower pomerons they do not cancel and generate a non-zero inclusive cross-section, corresponding to Fig. 2e. Below we list the analogous contributions from the rest of the terms in (14). The second term gives

$$2g^3\sqrt{2N_c}\,(P(3,4)P(1',2')V_lG(1',2'|1,2) - P(3',4')V_lG(3',4'|3,4)P(1,2)) \otimes W(1,2,4|1k,24-k)D_3^{(134)}(1k,3,24-k). \quad (52)$$

The third term gives

$$2g^3\sqrt{2N_c}\,(P(3,4)P(1',2')V_lG(1',2'|1,2) - P(3',4')V_lG(3',4'|3,4)P(1,2)) \otimes W(2,3,4|23k,4-k)D_3^{(124)}(1,23k,4-k) \quad (53)$$

and the last one

$$2g^3\sqrt{2N_c}\,(P(1,2)P(3',4')V_lG(3',4'|3,4) - P(1',2')V_lG(1',2'|1,2)P(3,4)) \otimes W(1,3,4|13k,4-k)D_3^{(124)}(13k,2,4-k). \quad (54)$$

Summing all the contributions and using the symmetries under $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$ we finally get the inclusive cross-section

$$I_1^{(Z)}(k,l) = 2g^4N_cP(3,4)P(1',2')V_lG(1',2'|1,2) \times \{W(1,2,3|1k,23-k)\,(D(4)-D(23-k)) - W(1,3,4|13k,4-k)\,(D(13k)-D(2))\} + (12 \leftrightarrow 34)\,. \quad (55)$$

Explicitly this cross-section has the form (passing to Φs)

$$I_1^{(Z)}(k,l) = 2g^2N_c\int\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_1'}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2} \times \Phi(3)\Phi(1'+l)V_lG(1'|1) \times\left\{\left(\frac{k_1^2}{k^2(k_1+k)^2} + \frac{k_3^2}{(k_1+k)^2(k_1+k-k_3)^2} - \frac{(k_1-k_3)^2}{(k_1+k)^2(k_1+k-k_3)^2}\right) \times\left(k_3^4P(k_3)-(k_1+k-k_3)^4P(k_1+k-k_3)\right) + (1\leftrightarrow 3)\right\} + \{12\leftrightarrow 34\}. \quad (56)$$

Thus we get a non-trivial inclusive cross-section corresponding to the diagram Fig. 2e which is naively prohibited by the AGK rules. Note that the emission from the vertex operator is found to be different from that in the diagram Fig. 2d allowed by the AGK rules.

5.2 Transition to the vertex V

As with the single inclusive cross-section we have to pass from the vertex Z to the vertex V in the triple pomeron diagrams Fig. 2a,b and also take into account the double pomeron exchange diagram Fig. 2c.

We begin with the diagram in Fig. 2a with both emissions from the upper pomeron. Transition to the vertex V here is achieved in full similarity with the single inclusive cross-section. The only difference in the derivation is that the uppermost pomeron has to be substituted according to (5). As a result we get an additional contribution for the emission from the vertex and upper pomeron (Fig. 2d), the new emission from the vertex described by the Kovchegov–Tuchin operator:

$$I^{(\mathrm{KT})}(k,l) = -\frac{g^2N_c}{\pi}\frac{1}{l^2}\int\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\frac{\mathrm{d}^2k_5}{(2\pi)^2} \times\Phi(1)\Phi(3)(k_1+k_3)^2(k_1+k_3+l)^2 \times G(13l|5)V_kP(5+k). \quad (57)$$

Now we pass to the diagram with two emissions from the lower pomerons, Fig. 2b. Here we use the fact that the gluon k has its rapidity y_1 larger than the rapidity y_2 of the gluon l. Accordingly we use the identity

$$G_{y-y_2}(3'',4''|3,4) = G_{y_1-y_2}(3'',4''|3',4')\otimes G_{y-y_1}(3',4'|3,4) \quad (58)$$

and present the part below the vertex Z at rapidity y in the form

$$P_{y_1}(1',2')V_kG_{y-y_1}(1',2'|1,2) \times P_{y_2}(3'',4'')V_lG_{y_1-y_2}(3'',4''|3',4') \otimes G_{y-y_1}(3',4'|3,4) = P_{y_1}(1',2')V_kP_{y_2}(3'',4'')V_lG_{y_1-y_2}(3'',4''|3',4') \otimes G_{4,y-y_1}(1',2',3',4'|1,2,3,4), \quad (59)$$

where G_4 is the Green function for the 4 gluons. After integration with the vertex and upper part we represent the diagram Fig. 2b as

$$P_{y_1}(1',2')V_kP_{y_2}(3'',4'')V_lG_{y_1-y_2}(3'',4''|3',4') \otimes G_{4,y-y_1}(1',2',3',4'|1,2,3,4) \otimes Z(1,2,3,4|\bar{1},\bar{4})D(\bar{1}). \quad (60)$$

Now we recall the basic relation between the vertices Z and V:

$$D_4 = G_4 \otimes D_4^{(0)} + G_4 \otimes Z \otimes D = D_4^{(\mathrm{R})} + G_4 \otimes V \otimes D, \quad (61)$$

from which we find

$$G_4 \otimes Z \otimes D = D_4^{(\mathrm{R})} + G_4 \otimes V \otimes D - G_4 \otimes D_4^{(0)}. \quad (62)$$

Putting this into (60) we find three terms for the double inclusive cross-section. The second one is just the desired structure with two emissions from the lower pomerons and vertex V:

$$P_{y_1}(1',2')V_k G_{y-y_1}(1'2'|1,2) \times P_{y_2}(3',4')V_l G_{y-y_2}(3',4'|3,4) \otimes V(1,2,3,4|\bar{1},\bar{4}) \otimes D(\bar{1}). \quad (63)$$

The third one corresponds to the emission from the double pomeron exchange with the minus sign:

$$- P_{y_1}(1',2')V_k P_{y_2}(3'',4'')V_l G_{y_1-y_2}(3'',4''|3',4') \quad (64)$$
$$\otimes G_{4,Y-y}(1',2',3',4'|1,2,3,4) \otimes D_4^{(0)}(1,2,3,4).$$

This contribution will cancel the same contribution from the diagram Fig. 2c.

Finally from the first term we shall have a new contribution to the emission from the vertex and a lower pomeron (Fig. 2e) with the structure

$$P_{y_1}(1,2)V_k P_{y_2}(3',4')V_l G_{y_1-y_2}(3',4'|3,4) \otimes D^{(\mathrm{R})}_{Y-y_1}(1,2,3,4). \quad (65)$$

This contribution is subdomimant at high energies, since the term $D^{(\mathrm{R})}$ in (62) is obviously subdominant relative to the other two. So strictly speaking we have to drop it just as we have done with a similar term for the single inclusive cross-section (cf. (33)). However, it grows faster than the contributions coming from the emissions from the upper pomeron or the upper pomeron and the vertex (Fig. 2a,d). So we study this term in some more detail, in particular because it has certain peculiarities.

Using the explicit expression of $D^{(\mathrm{R})}$ [9,12], (65) generates the cross-section

$$I_2(k,l) = -\frac{1}{2} g^2 P_{y_1}(1,2)V_k P_{y_2}(3',4')V_l G_{y_1-y_2}(3',4'|3,4) \otimes \left(\sum_{i=1}^{4} D_{Y-y_1}(i) - \sum_{i=2}^{4} D_{Y-y_1}(1i) \right). \quad (66)$$

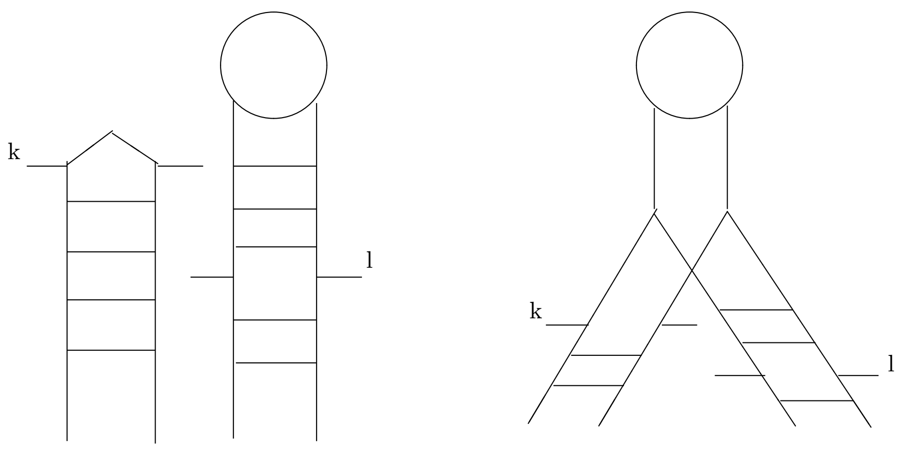
All terms depending on the sum 34 give zero, since the Green function $G(3',4'|3,4)$ vanishes when the two reggeons are located at the same point in the configuration space. Using the symmetry in $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$ we get

$$I_2(k,l) = -g^2 P_{y_1}(1,2)V_k P_{y_2}(3',4')V_l G_{y_1-y_2}(3',4'|3,4) \otimes \left(D_{Y-y_1}(3) - D_{Y-y_1}(13) \right). \quad (67)$$

The first term gives (passing to Φs)

$$I_2^{(1)}(k,l) = -\frac{g^2 N_c}{\pi k^2} \int \frac{\mathrm{d}^2 k_1}{(2\pi)^2} \Phi_{y_1}(1) \frac{k_1^2}{(k_1-k)^2} \times \int \frac{\mathrm{d}^2 k_3'}{(2\pi)^2} \frac{\mathrm{d}^2 k_3}{(2\pi)^2} \times \Phi_{y_2}(3') V_l G_{y_1-y_2}(3'|3) k_3^4 P_{Y-y_1}(3). \quad (68)$$

The integral over k_3 and k_3' is just 1/2 of the single inclusive cross-section from a single pomeron exchange $J(l)$. The first factor corresponds to the lower pomeron "opened" at the top and integrated over the topmost gluons. Such a contribution does not look very natural (see Fig. 7a). Moreover the integral over k_1 obviously diverges at $k_1 = k$. Of course this divergence is in fact spurious and arises only because we have neglected the momentum transfer in the target, taking it at $t = 0$. Its appearance is a signal that this approximation is too crude: in fact the first factor behaves as $-(1/y_1)\log t$ as $t \to 0$. Integration with the nuclear wave function will convert this behavior into a factor proportional to $(1/y_1)\log A$.



a b

Fig. 7. Illustration of terms $I_2^{(1)}$ and $I_2^{(2)}$; see (68) and (69)

The second term in (67) gives a non-factorizable integral:

$$I_2^{(2)}(k,l) = \frac{g^2 N_c}{\pi k^2} \int \frac{\mathrm{d}^2 k_1}{(2\pi)^2} \frac{\mathrm{d}^2 k_3'}{(2\pi)^2} \frac{\mathrm{d}^2 k_3}{(2\pi)^2} \times \Phi_{y_1}(1) \Phi_{y_2}(3') V_l G_{y_1-y_2}(3'|3) \frac{k_1^2 (k_1+k_3)^4}{(k_1-k)^2} \times P_{Y-y_1}(13). \quad (69)$$

Its structure is shown in Fig. 7b. It also diverges at $k_1 = k$ and so leads to terms proportional to $(1/y_1)\log A$.

6 Conclusions

We have studied the single and double inclusive cross-sections by inspection of the reggeized gluon diagrams in the Lipatov–Bartels formalism, as an alternative to the color dipole approach employed in [4,7]. As a result we have found the cross-sections to be much more complicated than obtained both in the latter approach and by the naive application of the AGK rules in [3]. Various terms corresponding to the emission of gluon from the triple pomeron vertex have been discovered. Among them the term derived in [4], which in their approach comes by the change from the quark dipole to the gluon dipole, in our treatment emerges as a result of transition from the original diffractive vertex Z to the effective vertex V used in the BK equation. However it does not exhaust all the contributions to the emission from the vertex, which are numerous and rather complicated in structure. In correspondence with the AGK rules we have no contribution from emissions below the splitting vertex. This cancellation was also found in the dipole picture in [4]. In the new contribution from the vertex we also discovered certain cancellations, which leave essentially only the diffractive part. This cancellation does not seem to have a counterpart in the dipole formalism, where all these new terms seem to be absent and in fact emission from the splitting point does not seem to appear at all.

In the double inclusive cross-section we have found terms which correspond to the emission of one gluon from the vertex and the other from one of the two pomerons immediately below the vertex. Such terms are not expected by the AGK rules provided the cut vertex does not depend on the way it is cut, which is usually assumed to be valid. Our results show that the latter assumption is not so. So while strictly speaking one cannot state that the AGK rules are violated, their naive application does not seem to hold. Still it is to be noted that we have not found contributions corresponding to emission from pomerons both above and below the vertex, as in [7], which would indeed strongly violate the AGK rules. Also all found terms in the double inclusive cross-section with a somewhat unexpected structure are subdominant at high energies and their inclusion seems to be questionable.

We have to acknowledge that our derivation has been rather heuristic. It has been based on the study of reggeized gluon diagrams for the triple discontinuity of the 4-gluon amplitude. Analysis of the gluon content of the triple gluon vertex has been performed using its diagrammatic representation, by the visual inspection of the way the real gluons are cut in the vertex. This ingredient of our approach has not been rigorously proven, that is, interpreted in terms of the relevant production amplitudes. So one has to take our derivation with a certain dose of caution. The complicated character of the contributions to emission from the vertex does not correspond to our expectations for something so fundamental. However it does not seem that one can avoid certain extra terms, apart from the easily located KT term, for emission from the vertex, since already in the high-mass diffraction such terms obviously appear [9].

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Appendix A: Inclusive cross-section from a single pomeron exchange

In this appendix we derive the inclusive cross-section for single pomeron exchange with emphasis on the numerical coefficient, important for the comparison with other contributions. We follow the normalizations of Lipatov in [13]. Our basic equations will be the unitarity relation for the reggeized gluon ladder together with the form of the multi-Regge amplitude for production of n real gluons. As compared to [13] we make the following changes. We lift the integration over the last reggeized gluon attached to the target (momentum q_{n+1}) and consider the amplitude at fixed q_{n+1}. Then the number of momentum integrations n will match the number of produced real gluons in the intermediate states. This allows one to include each factor $1/(4\pi)$ in the phase volume into the gluon interaction substituting in it $g^2 \to g^2/(2\pi)$ and absorbing $1/2$ into the sum over polarizations:

$$(1/2)\sum_\mu C_\mu(q_1,q_2) C^\mu(q_1,q_2) = U(q_1,q_2). \quad (A.1)$$

With these assumptions, the forward unitarity relation becomes

$$\mathrm{Im}\, A_n = \frac{\pi}{s} g^2 \left(\frac{g^2 N_c}{2\pi}\right)^n \times \int \prod_{i=1}^{n} \frac{\mathrm{d}^2 q_i}{(2\pi)^2} \int \prod_{i=1}^{n+1} \mathrm{d}s_{i,i-1} \delta\left(\prod_{i=1}^{n+1} s_{i,i-1} - s\prod_{i=1}^{n} p_i^2\right) \times |B_n(q_i)|^2, \quad (A.2)$$

where the intermediate real gluon have momenta $p_i = q_i - q_{i+1}$, $i = 1, n$ and the production amplitude B_n has the multi-Regge form:

$$B_n = 2s \frac{s_{n+1,n}^{\omega_{n+1}}}{t_{n+1}} \prod_{i=1}^{n} \frac{s_{i,i-1}^{\omega_i}}{t_i} \left(e_i C(q_{i+1},q_i)\right). \quad (A.3)$$

Here $t_i = -q_i^2$, e_i are the gluon polarization vectors, $\omega_i = \omega(t_i)$ and $\omega(t)$ is the gluon Regge trajectory which can be found in [13] together with the explicit expression for vectors C. In (A.2) and (A.3) we suppressed all the color indices, summation over which is done in the standard way.

Our aim is to factorize the unitarity relation into the product of two independent ones for parts of the ladder above and below some particular real gluon with momentum p_m. To this end we first factorize the production amplitudes introducing

$$B^{(1)}_{m-1} = 2s_1 \frac{s^{\omega_m}_{m,m-1}}{t_m} \prod_{i=1}^{m-1} \frac{s^{\omega_i}_{i,i-1}}{t_i} \left(e_i C(q_{i+1}, q_i)\right) \qquad (A.4)$$

and

$$B^{(2)}_{n-m} = 2s_1 \frac{s^{\omega_{n+1}}_{n+1,n}}{t_{n+1}} \prod_{i=m+1}^{n} \frac{s^{\omega_i}_{i,i-1}}{t_i} \left(e_i C(q_{i+1}, q_i)\right), \qquad (A.5)$$

with $s_1 s_2 = s p_m^2$. Obviously

$$B_n = \frac{s}{2s_1 s_2} B^{(1)}_{m-1} B^{(2)}_{n-m} \left(e_m C(q_{m+1}, q_m)\right). \qquad (A.6)$$

Integration over the intermediate momenta of the square moduli of the amplitudes $B^{(1)}_{m-1}$ and $B^{(2)}$ generates unitarity relations for the corresponding elastic amplitudes:

$$\mathrm{Im}\, A^{(1)}_{m-1} = \frac{\pi}{s_1} g^2 \left(\frac{g^2 N_c}{2\pi}\right)^{m-1} \times \int \prod_{i=1}^{m-1} \frac{\mathrm{d}^2 q_i}{(2\pi)^2} \int \prod_{i=1}^{m} \mathrm{d}s_{i,i-1} \delta\left(\prod_{i=1}^{m} s_{i,i-1} - s_1 \prod_{i=1}^{m-1} p_i^2\right) \times |B^{(1)}_{m-1}(q_i)|^2 \qquad (A.7)$$

and

$$\mathrm{Im}\, A^{(2)}_{n-m} = \frac{\pi}{s} \left(\frac{g^2 N_c}{2\pi}\right)^{n-m} \times \int \prod_{i=m+2}^{n} \frac{\mathrm{d}^2 q_i}{(2\pi)^2} \int \prod_{i=m+1}^{n+1} \mathrm{d}s_{i,i-1} \times \delta\left(\prod_{i=m+1}^{n+1} s_{i,i-1} - s_2 \prod_{i=m+1}^{n} p_i^2\right) |B^{(2)}_{n-m}(q_i)|^2. \qquad (A.8)$$

Now we turn to the overall unitarity relation (A.2). We represent it in the following form:

$$\delta\left(\prod_{i=1}^{n+1} s_{i,i-1} - s \prod_{i=1}^{n} p_i^2\right) = \int \mathrm{d}s_1 \mathrm{d}s_2 \delta(s_1 s_2 - s p_m^2) \times \delta\left(\prod_{i=1}^{m} s_{i,i-1} - s_1 \prod_{i=1}^{m-1} p_i^2\right) \times \delta\left(\prod_{i=m+1}^{n+1} s_{i,i-1} - s_2 \prod_{i=m+1}^{n} p_i^2\right). \qquad (A.9)$$

Then using the unitarity relations (A.7) and (A.8) and summing over the polarizations of the distinguished gluon p_m we obtain

$$\mathrm{Im}\, A_n = \frac{g^2 N_c}{8\pi^2} \int \mathrm{d}s_1 \mathrm{d}s_2 \frac{s}{s_1 s_2} \times \int \frac{\mathrm{d}^2 q_m}{(2\pi)^2} \frac{\mathrm{d}^2 q_{m+1}}{(2\pi)^2} \delta(s_1 s_2 - s p_m^2) \mathrm{Im}\, A^{(1)}_{m-1} \mathrm{Im}\, A^{(2)}_{n-m} \times U(q_{m+1}, q_m). \qquad (A.10)$$

In the Lipatov normalization

$$\mathrm{Im}\, A = 2sP. \qquad (A.11)$$

So relation (A.10) in terms of pomerons acquires the form

$$P_n = \frac{g^2 N_c}{4\pi^2} \int \mathrm{d}s_1 \mathrm{d}s_2 \int \frac{\mathrm{d}^2 q_m}{(2\pi)^2} \frac{\mathrm{d}^2 q_{m+1}}{(2\pi)^2} \delta(s_1 s_2 - s p_m^2) \times P_{m-1}(q_m) P_{n-m}(q_{m+1}) U(q_{m+1}, q_m). \qquad (A.12)$$

The inclusive cross-section results by summing over all numbers of gluons in the two pomerons on the right-hand side of (A.12), fixing momentum $p_m = k$ and doubling the contribution according to (A.11)

$$\mathrm{d}\sigma = 4\alpha_{\mathrm{s}} N_c \int \mathrm{d}s_1 \mathrm{d}s_2 \frac{\mathrm{d}^2 k}{(2\pi)^3} \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \delta(s_1 s_2 - s k^2) \times P(q) P(q-k) U(q-k, q). \qquad (A.13)$$

Thus the final recipe for the inclusive cross-section is to introduce the operator

$$V_k = 2\alpha_{\mathrm{s}} N_c U(q', q) \delta^2(q - q' - k), \qquad (A.14)$$

multiply the result by 2, and the resulting cross-section is

$$I(k) = \frac{(2\pi)^3 \mathrm{d}\sigma}{\mathrm{d}y \mathrm{d}^2 k}. \qquad (A.15)$$

Appendix B: Explicit expressions for $I^{(Z)}$

To find the explicit expressions for the inclusive cross-section $I^{(Z)}(k)$ given by (25) we shall use the following relations between the pomerons $P(k)$, pomerons in the nucleus $\Phi(k)$, the function $\phi(k)$ which satisfies the non-linear BFKL equation and the gluon density defined as $h(k) = k^2 \nabla_k^2 \phi(k)$:

$$g^2 P(k) \to \Phi(k) = -2\pi \nabla_k^2 \phi(k) = (2\pi)^2 \delta^2(k) - 2\pi \frac{h(k)}{k^2}, \qquad (B.1)$$

$$P(k) = -2\pi \frac{h_0(k)}{k^2}. \qquad (B.2)$$

We shall also use $\Phi(r=0) = P(r=0) = 0$ in the configuration space and the relation

$$\Delta \ln r = 2\pi \delta^2(r). \qquad (B.3)$$

Armed with these relations we study the separate terms in (25). The first one (with $D(3-k)$) has the form

$$I_1^{(Z)} = g^2 N_c \times \int \frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\Phi(k_1)\Phi(k_3)P(k_3-k)(k_3-k)^4 \times \left\{\frac{k_1^2}{k^2(k_1-k)^2}+\frac{k_3^2}{k^2(k_3-k)^2} - \frac{(k_1-k_3)^2}{(k_1-k)^2(k_3-k)^2}\right\}. \quad \text{(B.4)}$$

The bracket vanishes if $k_1 = 0$ or $k_3 = 0$. So in principle we can substitute Φs and P by gluon densities neglecting the δ term in (B.1). However for this particular term this is not convenient. The second term in the brackets gives zero, since integration over k_1 then leads to $\Phi(r_1 = 0)$. The other two terms can be rearranged to exhibit the absence of an infrared divergence to obtain

$$I_1^{(Z)} = g^2 N_c \times \int \frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\Phi(k_1)\Phi(k_3)P(k_3-k)(k_3-k)^2 \times \left\{\frac{k_1^2-k^2}{k^2(k_1-k)^2}+2\frac{(\boldsymbol{k}_1-\boldsymbol{k})(\boldsymbol{k}_3-\boldsymbol{k})}{(k_1-k)^2(k_3-k)^2}\right\}. \quad \text{(B.5)}$$

Each of the two terms obviously factorizes in two integrals over k_1 and k_3.

The first term in the bracket contains two integrals:

$$\int \frac{\mathrm{d}^2k_1}{(2\pi)^2}\Phi(k_1)\frac{k_1^2-k^2}{k^2(k_1-k)^2} = \frac{2}{k^2}\boldsymbol{k}\nabla_k\phi(k) = \frac{2}{k}\phi'(k) \quad \text{(B.6)}$$

and

$$\int \frac{\mathrm{d}^2k_3}{(2\pi)^2}\Phi(k_3)P(k_3-k)(k_3-k)^4 = \int \mathrm{d}^2r\mathrm{e}^{\mathrm{i}kr}\Phi(r)\nabla^4P(r) \equiv X_1(k). \quad \text{(B.7)}$$

In the second term the integral over k_1 can be represented by

$$\nabla_k \int \frac{\mathrm{d}^2k_1}{2\pi}\nabla_1^2\phi(k_1)\ln(k_1-k) = \nabla_k\phi(k), \quad \text{(B.8)}$$

where we have integrated by parts and used the relation (B.3). The integral over k_3 can be written as

$$-\frac{1}{4}\int \frac{\mathrm{d}^2k_3}{(2\pi)^2}\Phi(k_3)P(k_3-k)\nabla_k(k_3-k)^4 = \frac{1}{4}\int \mathrm{d}^2r\boldsymbol{r}\sin\boldsymbol{k}r\Phi(r)\nabla^4P(r) \equiv \boldsymbol{k}X_2(k). \quad \text{(B.9)}$$

Our final expression for this part is

$$I_1^{(Z)}(k) = 2g^2N_c\phi'(k)\left(\frac{1}{k}X_1(k)+kX_2(k)\right). \quad \text{(B.10)}$$

The second term (with $D(4)$) differs from the first by the sign and the change $(k_3-k)^4P(k_3-k) \to k_3^4P(k_3)$. Obviously the factorization is preserved with the integrals over k_3 changed. Now

$$\int \frac{\mathrm{d}^2k_3}{(2\pi)^2}\Phi(k_3)P(k_3)k_3^4 = \int \mathrm{d}^2k_3h(k_3)h_0(k_3) = \int \mathrm{d}^2r\Phi(r)\nabla^4P(r) \equiv X_3 \quad \text{(B.11)}$$

(it does not depend on k) and we have the second integral

$$-\nabla_k\int \frac{\mathrm{d}^2k_3}{(2\pi)^2}k_3^4\Phi(k_3)P(k_3)\ln(k_3-k) = -\nabla_k\int \mathrm{d}^2k_3h(k_3)h_0(k_3)\ln(k_3-k) \equiv \boldsymbol{k}X_4(k). \quad \text{(B.12)}$$

So we find

$$I_2^{(Z)}(k) = -2g^2N_c\phi'(k)\left(\frac{1}{k}X_3+kX_4(k)\right). \quad \text{(B.13)}$$

The third and fourth terms (with $D(2k)$ and $D(1)$) differ from the first two ones by the interchange of the two lower pomerons and the sign of k. Since the cross-section is obviously independent of this sign and symmetric in the lower pomerons, the third and fourth terms give the same contribution as the first two, so that (B.10) and (B.13) have to be doubled.

The fifth term (with $D(12k)$) is

$$I_5^{(Z)}(k) = g^2N_ck^4P(k)\int \frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\Phi(1)\Phi(3) \times \left\{\frac{k_1^2-k^2}{k^2(k_1-k)^2}+\frac{k_3^2-k^2}{k^2(k_3-k)^2} + 2\frac{(\boldsymbol{k}_1-\boldsymbol{k})(\boldsymbol{k}_3-\boldsymbol{k})}{(k_1-k)^2(k_3-k)^2}\right\}, \quad \text{(B.14)}$$

where we transformed the initial $W(1,23,4|12k,34-5)$ similarly to the transition from (B.4) to (B.5). The first two terms in the bracket do not give any contribution, so that we are left with the last one, which obviously factorizes in two identical integrals; see (B.8). So we find

$$I_5^{(Z)}(k) = 2g^2N_ck^4P(k)\left(\phi'(k)\right)^2. \quad \text{(B.15)}$$

Finally we have contributions from the sixth and seventh terms. They do not factorize and we just rewrite their sum in terms of h and h_0:

$$I_{6+7}^{(Z)}(k) = -\frac{g^2N_c}{2\pi}\int \frac{\mathrm{d}^2k_1}{k_1^2}\frac{\mathrm{d}^2k_3}{k_3^2}h(k_1)h(k_3) \times \left((k_1+k_3-k)^2h_0(k_1+k_3-k) - (k_1-k_3)^2h_0(k_1-k_3)\right) \times \left\{\frac{k_1^2-k^2}{k^2(k_1-k)^2}+\frac{k_3^2-k^2}{k^2(k_3-k)^2}\right.$$

$$+2\frac{(\boldsymbol{k}_1-\boldsymbol{k})(\boldsymbol{k}_3-\boldsymbol{k})}{(k_1-k)^2(k_3-k)^2}\Bigg\}. \tag{B.16}$$

The final inclusive cross-section $I^{(Z)}$ from the vertex Z is

$$I^{(Z)}=2I_1^{(Z)}+2I_2^{(Z)}+I_5^{(Z)}+I_{6+7}^{(Z)}. \tag{B.17}$$

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